Chapter 6

Verification of strength

'It is not possible to fight beyond your strength, even if you strive' – Homer (800 BC-700 BC)[1]

6.1 Basis of design

Verification of strength to Eurocode 7 involves checking that design effects of actions do not exceed their corresponding design resistances.

Verification of strength is expressed in Eurocode 7 by the inequality:

$$E_d \leq R_d$$ *[EN 1990 exp (6.8)]* & *[EN 1997-1 exp (2.5)]*

in which E_d = the design effects of actions and R_d = the corresponding design resistance.

This requirement applies to ultimate limit state GEO, defined as:

> *Failure or excessive deformation of the ground, in which the strength of soil or rock is significant in providing resistance* *[EN 1997-1 §2.4.7.1(1)P]*

and to ultimate limit state STR:

> *Internal failure or excessive deformation of the structure or structural elements … in which the strength of structural materials is significant in providing resistance* *[EN 1997-1 §2.4.7.1(1)P]*

Examples of situations where strength is a concern are shown in **Figure 79**; from left to right, these include: top, the stem of a cantilever retaining wall must withstand the forces on its back (STR); and a hillside must be strong enough to support its self-weight and other forces acting on it (GEO); middle, the foundation of a footing must be strong enough to support the imposed load on it (GEO); and an embedded retaining wall and its support system must be strong enough to withstand earth pressures over its retained height (STR); and bottom, the ground supporting a pile subject to horizontal loads must be strong enough to prevent excessive horizontal movement (GEO); and, finally, the ground beneath a mass concrete retaining wall must be strong enough to carry the wall's weight and any forces acting on it (GEO).

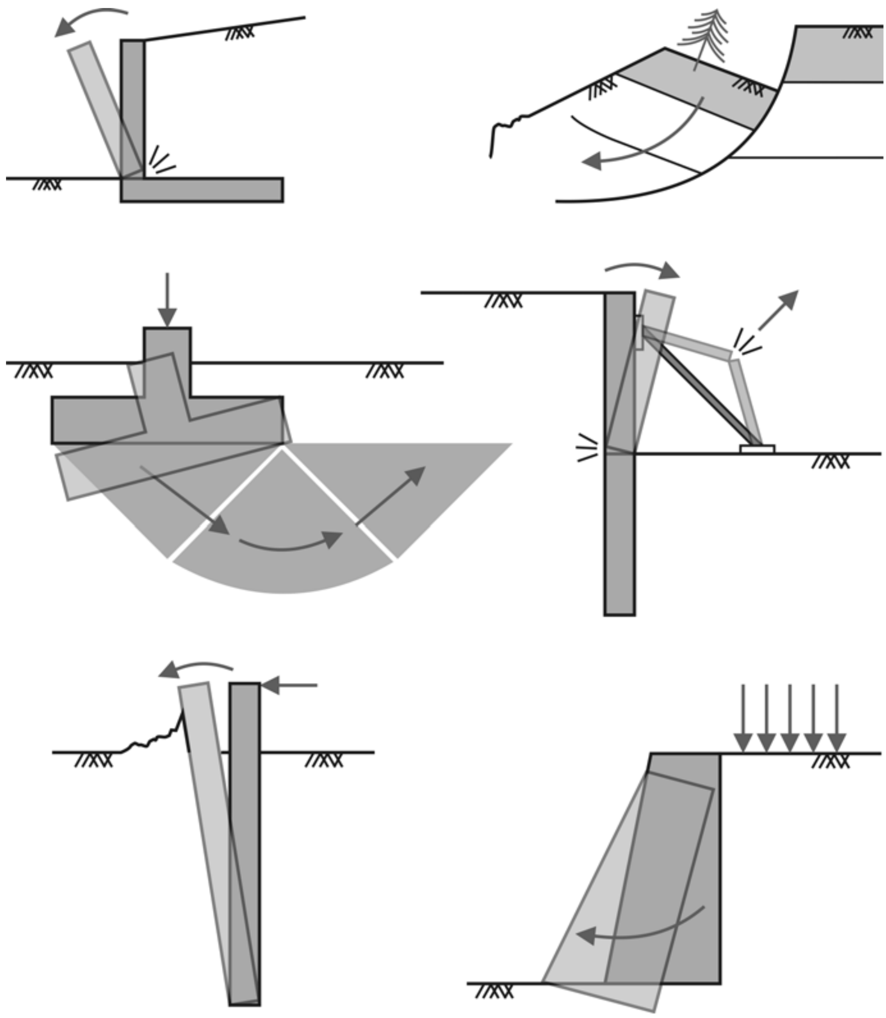

Figure 79. *Examples of ultimate limit states of strength*

6.1.1 Effects of actions

'Effects of actions' (or 'action effects') is a general term denoting internal forces, moments, stresses, and strains in structural members – plus the deflection and rotation of the whole structure. *[EN 1990 §1.5.3.2]*

For most structural designs, verification of limit state STR involves action effects that are independent of the *strength* of the structural materials (see Chapter 2). However, in many geotechnical designs, verification of the STR and GEO limit states involves effects of actions that depend upon the strength of the ground.

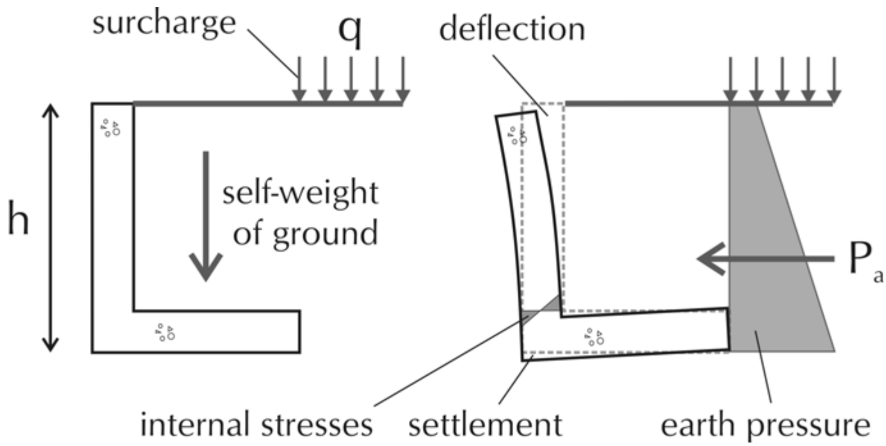

Figure 80*. Actions (left) and effects (right) for L-shaped gravity retaining wall*

For example, **Figure 80** shows a retaining wall supporting loose soil and an imposed uniform surcharge (q). The earth pressures acting behind the wall produce a horizontal sliding force H_E (an action effect) given by:

$$H_E = P_a = K_a\left(\frac{\gamma h}{2}+q\right)h = \left(\frac{1-\sin\phi}{1+\sin\phi}\right)\left(\frac{\gamma h}{2}+q\right)h = f\left\{h,\gamma,\phi,q\right\}$$

where h is the wall's height; γ and φ the soil's self-weight and angle of shearing resistance; and K_a is Rankine's active earth pressure coefficient.

This simple example illustrates why the definition of design effects of actions given in the head Eurocode:

$$E_d = E\left\{F_d; a_d\right\}$$ *[EN 1990 exp (6.2a, simplified)]*

has to be revised for geotechnical design to:

$$E_d = E\left\{F_d; X_d; a_d\right\}$$

where F_d = design actions applied to the structure; X_d = design material properties; and a_d = design dimensions of the structure. (The notation E{...} denotes a function of the enclosed parameters and usually involves multiple parameters of each type listed.)

Put simply, in structural design, effects of actions are generally a function of actions and dimensions only; whereas, in geotechnical design, effects of actions are typically a function of actions, dimensions, *and the strength of the ground*.

The inclusion of X_d in the equation for E_d adds considerable complexity to designs involving geotechnical actions and is one of the reasons for the diversity of design methods used in geotechnical design.

6.1.2 Resistance

'Resistance' is defined as the:

> *capacity of a [member or] component, or cross-section of a [member or] component of a structure, to withstand actions without mechanical failure*
>
> *[EN 1990 §1.5.2.15]* & *[EN 1997-1 §1.5.2.7]*

(The words in brackets are omitted in Eurocode 7's definition. The absence of the word 'ground' from either definition appears to be an error – unless we regard the ground as a component of the structure.)

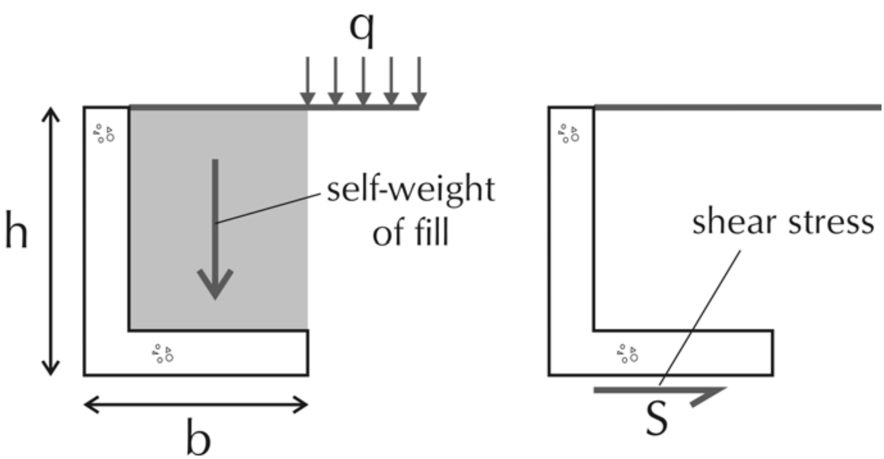

***Figure 81**. Sliding resistance of L-shaped gravity retaining wall*

For most structural designs, verification of limit state STR involves resistances that are independent of actions (see Chapter 2). However, in many geotechnical designs, verification of the STR and GEO limit states involves resistances that depend upon *actions*.

For example, **Figure 81** illustrates the sliding resistance H_R of the retaining wall shown previously in **Figure 80**:

$$H_R = S = \gamma \times h \times b \times \tan\delta = f\{h, b, \gamma, \delta\}$$

where the resistance is a function of the wall's dimensions (h and b), the self-weight of the soil (γ) – *and* the strength of the soil-structure interface (δ, which itself is a function of the soil's drained angle of shearing resistance φ).

Again this example illustrates why the definition of resistance given in the head Eurocode:

$$R_d = \frac{R\{X_d; a_d\}}{\gamma_{Rd}}$$ *[EN 1990 exp (6.6, simplified)]*

has to be revised for geotechnical design to:

$$R_d = \frac{R\{F_d; X_d; a_d\}}{\gamma_R}$$

where F_d, X_d, and a_d are as defined earlier. Here, R{...} denotes a function of the enclosed parameters and $\gamma_{Rd} = \gamma_R$ = a partial factor on resistance.

In simple terms, in structural design, resistances are generally a function of material strengths and dimensions only; whereas, in geotechnical design, resistances are typically a function of material strengths, dimensions, *and actions, including the self-weight of the ground.*

Once again, the inclusion of F_d in the equation for R_d adds considerable complexity to designs involving geotechnical materials and is another reason for the diversity of design methods used in geotechnical design.

6.2 Introducing reliability into the design

'The word safety is encompassed in the Eurocodes in the word reliability'[2]

Reliability can be introduced into the design in a number of ways, through the application of suitable partial factors or tolerances, as illustrated in **Figure 82**.

In the top half of this diagram, there are three 'channels' that lead into the calculation model: one for actions (left), another for geometrical parameters (centre), and a third for materials properties (right). Certain material properties, such as weight density, have a direct influence on actions, whereas other material properties, such as strength, do not (they do, however, influence the action effects).

Verification occurs in the bottom third of the diagram: the calculation model provides values for effects of actions (left) and resistance (right), which are compared against each other (in the centre).

Partial factors (or tolerances) can be applied to one or more of:

- actions (F) or action effects (E)
- material properties (X) and/or resistances (R)
- geometrical parameters (a)

These factors/tolerances are shown on **Figure 82** in the wavy boxes.

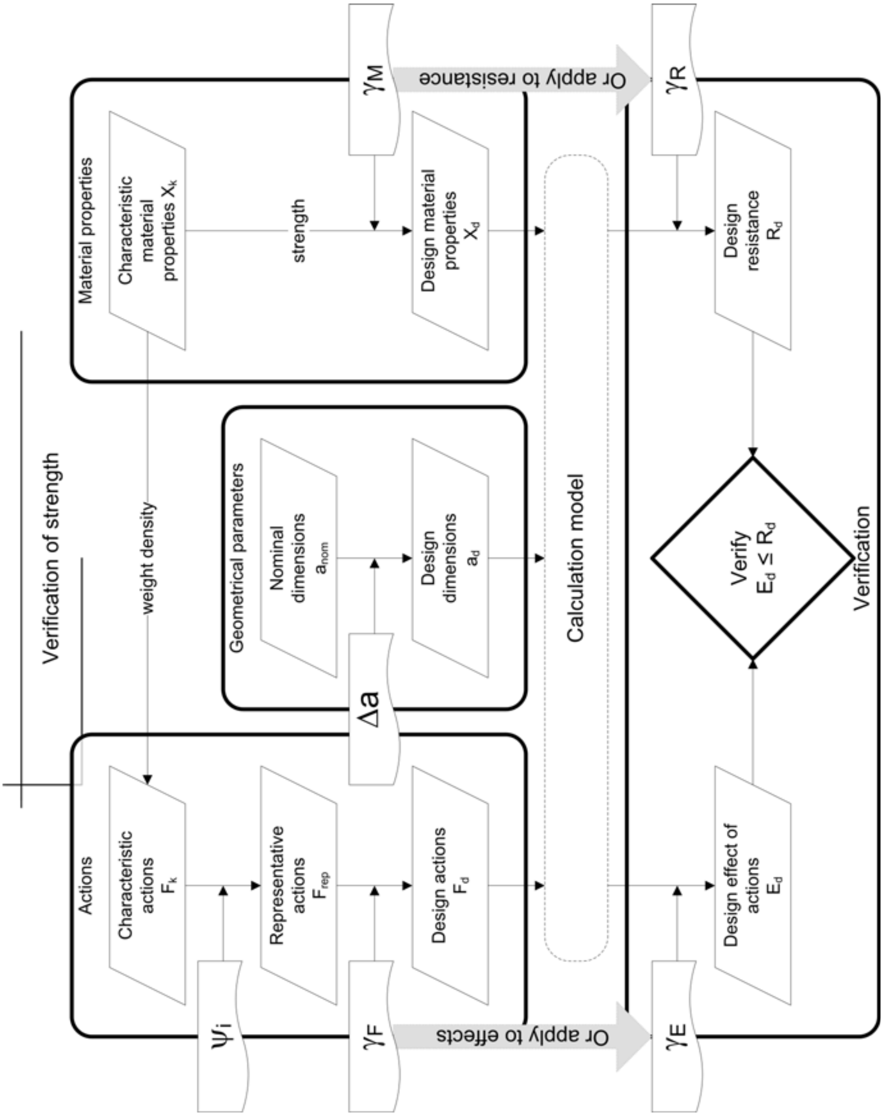

***Figure 82**. Overview of verification of strength*

6.2.1 Actions and effects

The calculation of design effects of actions follows the route shown in the left hand channel of **Figure 82**:

Characteristic actions → Representative actions → Design actions
→ Design effects of actions

Characteristic actions F_k are calculated according to the rules of Eurocode 1. Characteristic self-weights are calculated as the product of a material's characteristic weight density γ_k and its nominal dimensions a_{nom} (see Chapter 2):

$$F_k = \gamma_k \times a_{nom,1} \times a_{nom,2} \times a_{nom,3}$$

Representative actions F_{rep} are obtained from characteristic actions by multiplying by correlation factors $\psi \leq 1.0$ (where $\psi = 1.0$ for permanent actions, see Chapter 2):

$$F_{rep} = \psi F_k$$

The total design action F_d is then obtained as the sum of all the representative actions multiplied by their corresponding partial factors $\gamma_F \geq 1.0$:

$$F_d = \sum_i \gamma_{F,i} \psi_i F_{k,i}$$

The design effects of actions are then obtained from:

$$E_d = E\{F_d; X_d; a_d\} = E\{\gamma_F \psi F_k; X_d; a_d\}$$

Figure 83 shows the relative magnitude of actions as appropriate combination factors (1.0 or ψ) and partial factors (γ_G and γ_Q) are applied to them. The diagram assumes arbitrary values for the permanent, leading variable, and accompanying variable actions (G, Q_1, and Q_i respectively). The arrow denotes where design actions enter the calculation model.

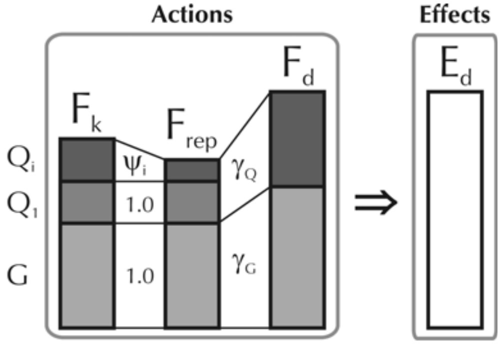

Figure 83. *Hierarchy of actions and effects when partial factors are applied to actions*

Eurocode 7 allows partial factors γ_F to be applied to actions or to their effects, *but typically not to both*. Thus an alternative to the above equation is:

$$E_d = \gamma_E E\left\{\psi F_k; X_d; a_d\right\}$$

where the partial factors γ_E are numerically identical to γ_F.

Figure 84 shows the relative magnitude of the actions and effects, as the combination factors are applied to characteristic actions and partial factors to the *effects* of actions. If the calculation model is linear, then the resultant design effects will be identical to those shown in **Figure 83**; if the model is non-linear (which is invariably the case in geotechnical engineering), then the resultant design effects will differ. A further complication with this formulation is that permanent and variable effects of actions must be calculated separately to allow different partial factors to be applied to them. Existing computer software is unlikely to have been programmed to do this and hence will need amending to accommodate Eurocode 7.

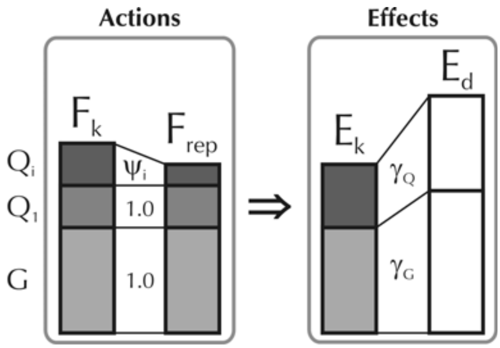

Figure 84. *Hierarchy of actions and effects when partial factors are applied to action effects*

6.2.2 Material strength and resistance

The calculation of design resistance follows the route shown in the right hand channel of **Figure 82**:

Characteristic material strengths → Design strengths
→ Design resistance

Design material properties X_d are obtained from characteristic material properties X_k by dividing by partial factors $\gamma_M \geq 1.0$:

$$X_d = \frac{X_k}{\gamma_M}$$

The design resistance is then obtained from:

$$R_d = \frac{R\{F_d;X_d;a_d\}}{\gamma_R} = \frac{R\left\{F_d;\dfrac{X_k}{\gamma_M};a_d\right\}}{\gamma_R}$$

where the partial factor $\gamma_R \geq 1.0$.

It is usual for one of the partial factors γ_M or γ_R to be equal to 1.0 and so the equation above typically reduces to one of two formats, either:

$$R_d = R\left\{F_d;\frac{X_k}{\gamma_M};a_d\right\} \text{ or } R_d = \frac{R\{F_d;X_k;a_d\}}{\gamma_R}$$

Figure 85 shows the relative magnitude of material strengths, assuming the first format, as the appropriate partial factors (γ_φ and γ_{cu}) are applied to them. The diagram assumes arbitrary contributions to resistance from a coarse soil with characteristic angle of shearing resistance φ_k and from a fine soil with characteristic undrained shear strength c_{uk}. The arrow denotes the insertion of design material strengths into the calculation model.

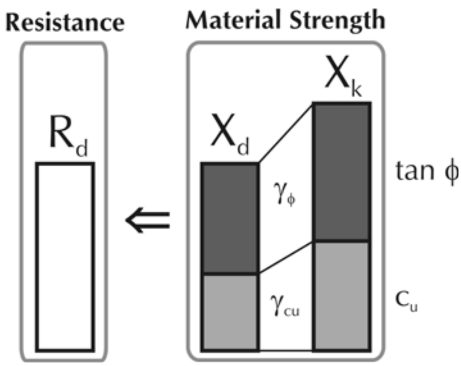

__Figure 85__. Hierarchy of material strengths and resistance when factors are applied to material properties only

Figure 86 does likewise for the second format, applying resistance factors (γ_R) instead of material factors. The arrow denotes the insertion of design material strengths into the calculation model. The resultant design resistance R_d will invariably differ from that shown in **Figure 85**.

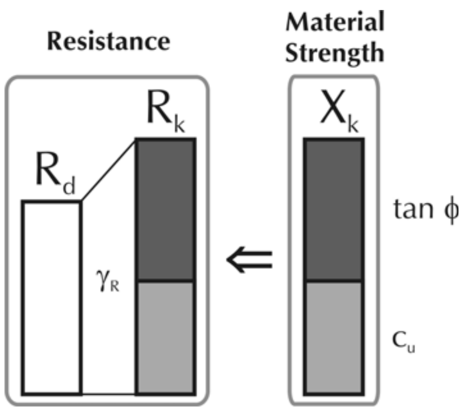

__Figure 86__. Hierarchy of material strengths and resistance when factors are applied to resistance only